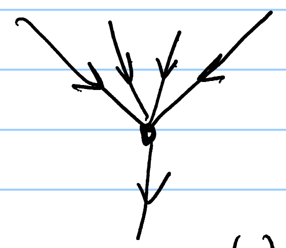
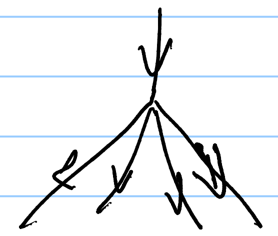
The Definition of "Quantum Group".

Definition A Quantum Group is a solution (in an FTS, "formal tensor space") of some equations for some specific elements m (really, (m_k)), Δ (really, (Δ_k)) and R.

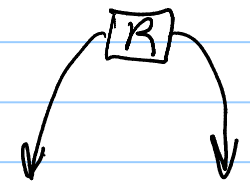
The elements:

m_k: 

(k in, 1 out)

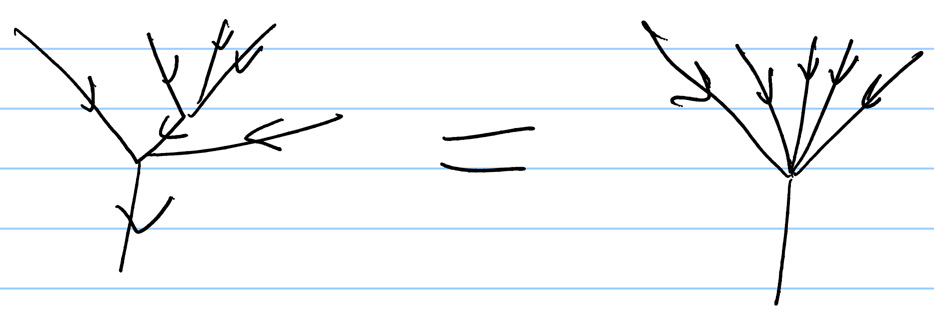
Δ_k: 

(1 in, k out)

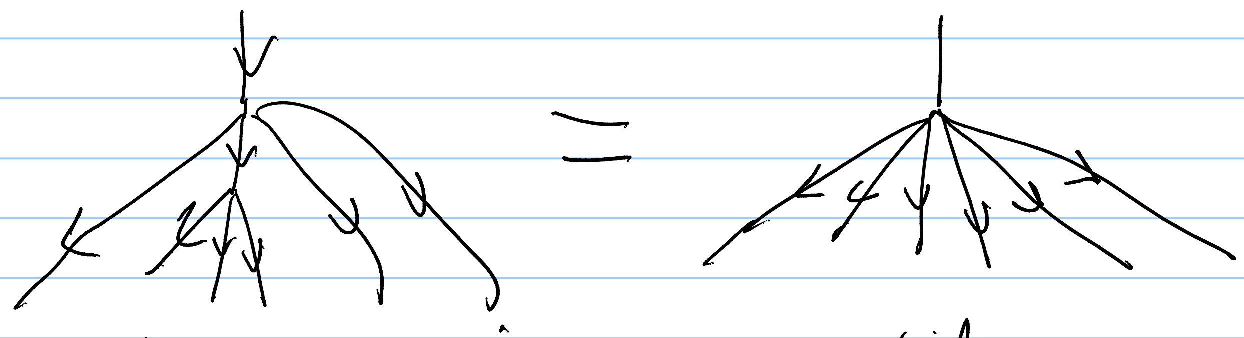
R: 

The equations:

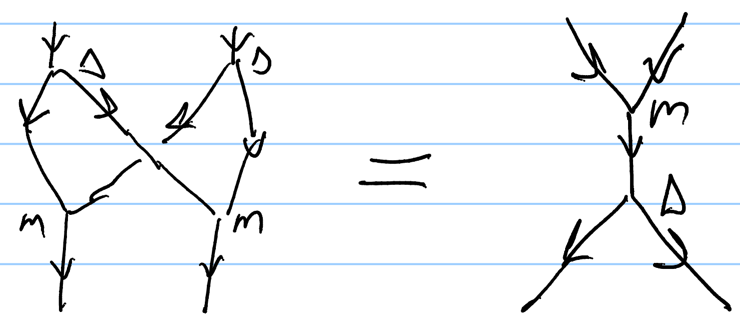
1. The m_k's define an "associative algebra":



2. The Δ_k's likewise define a "co-associative algebra":

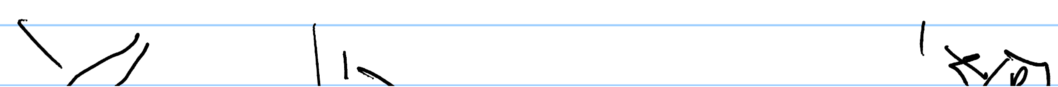


3. The two are Hopf-compatible:

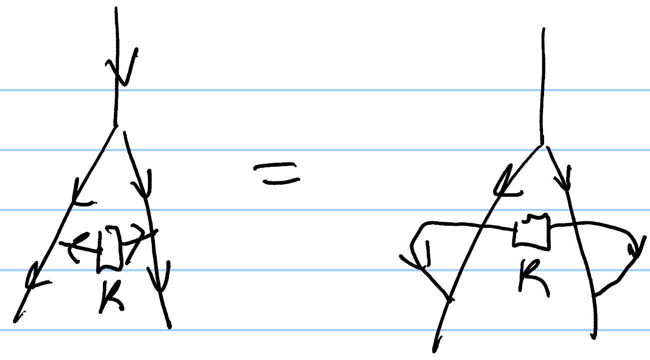


(what is the general K version of this?)

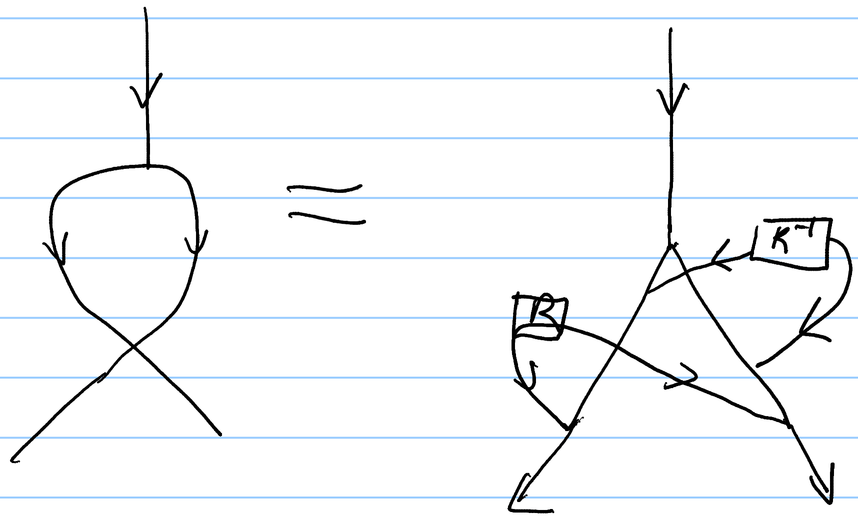
4. $(\Delta\otimes 1)R = R^{23}R^{13}$:



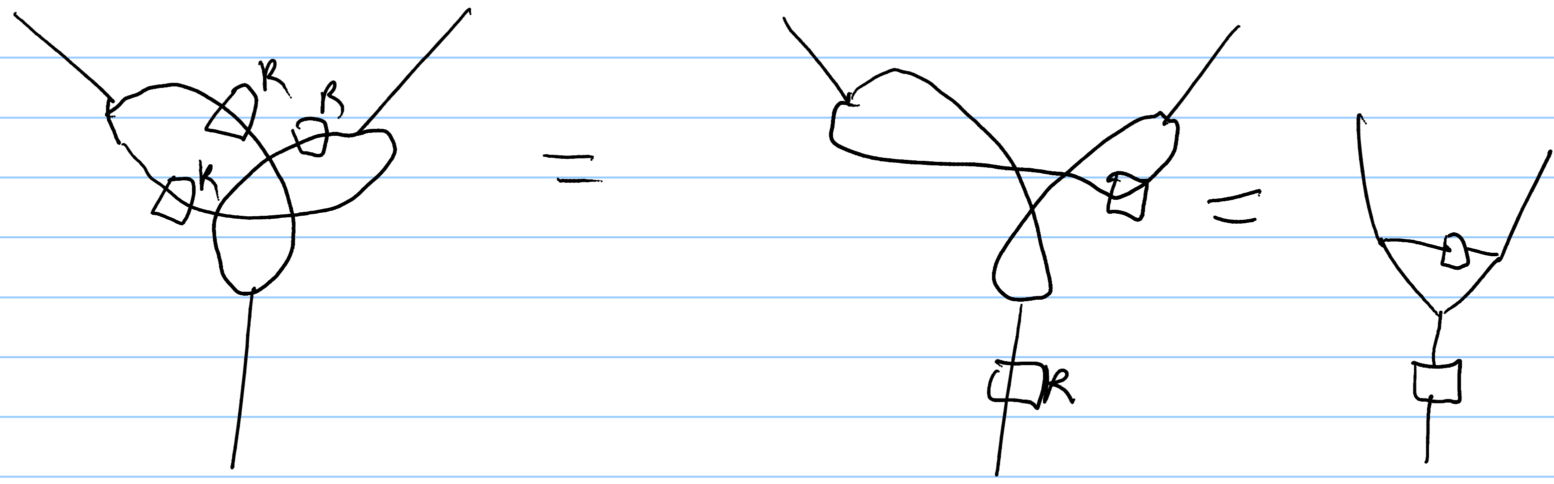
5. R commutes with $\Delta(a)$:



6 An equation meaning $\Delta^{op} = R\Delta R^{-1}$:



R3 is roughly



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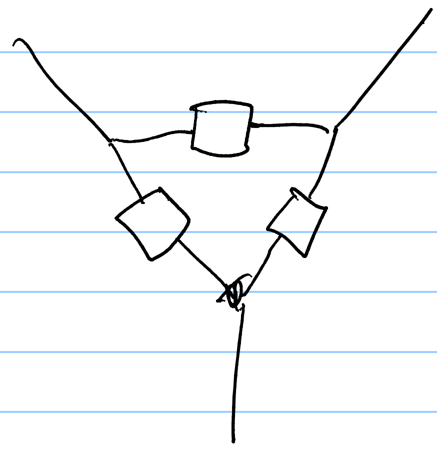
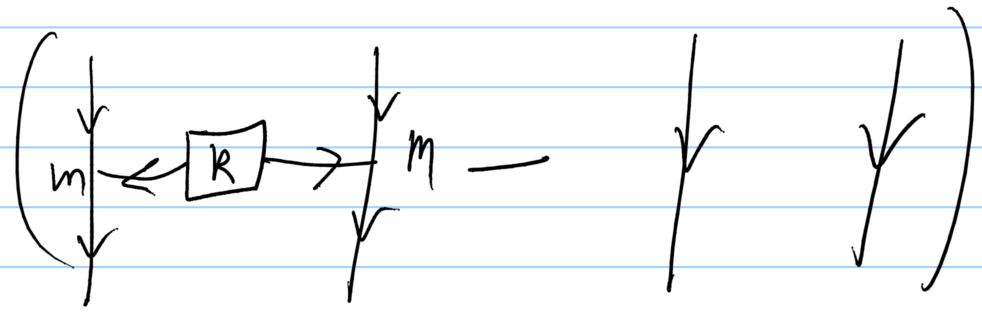
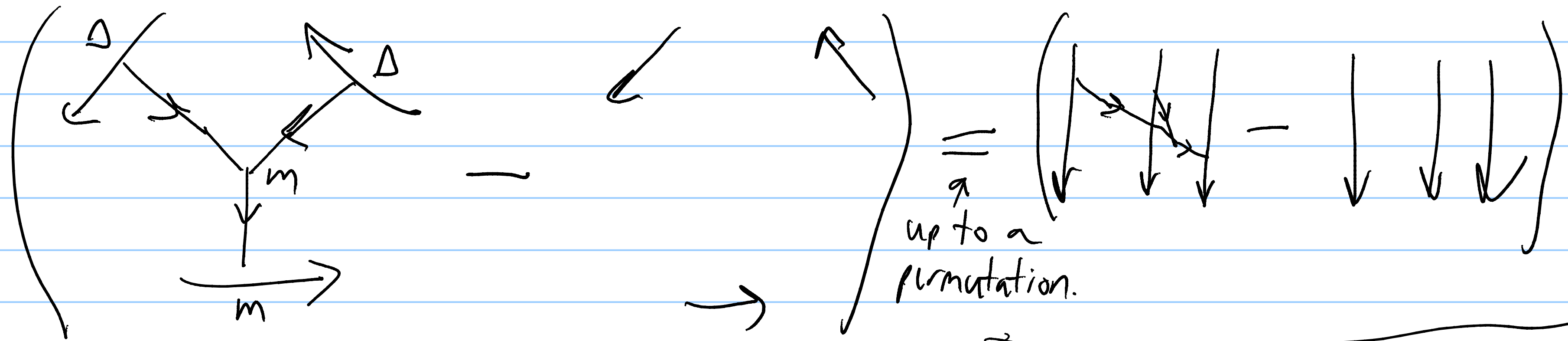


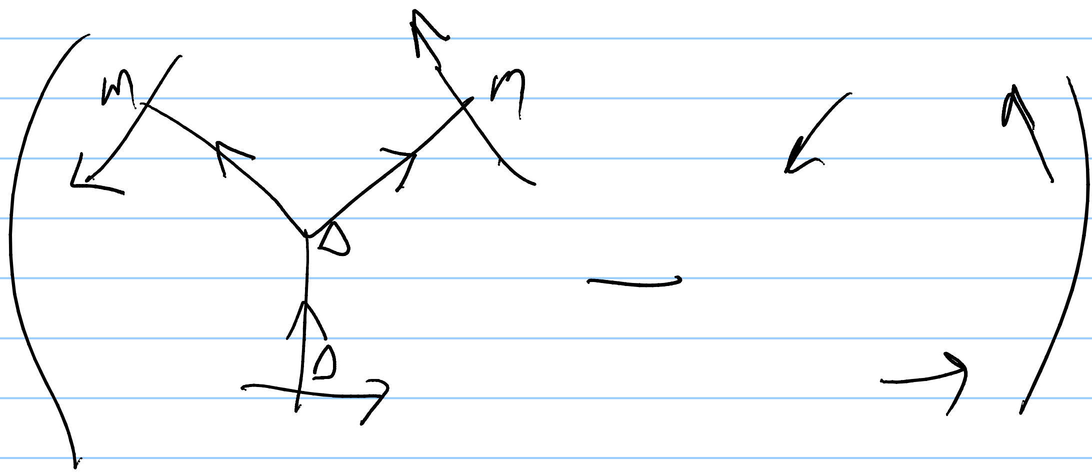
<u>Question</u> What is the "finite type condition" of the "free quantum group"? Are there expansions?

Examples of formal differences:

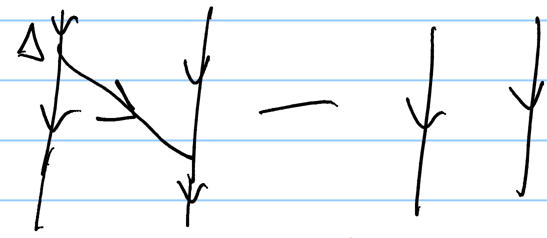


This must be "The arrow"

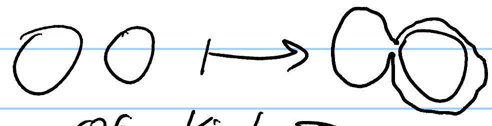




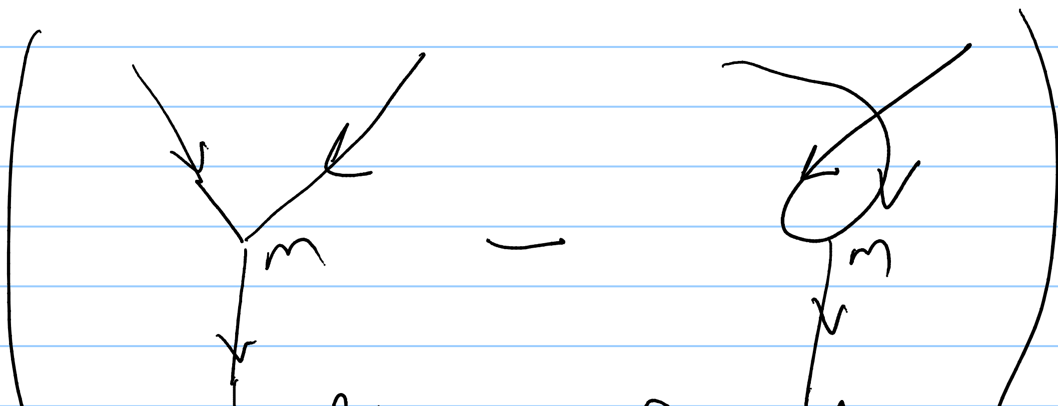
Also:



In knot theory this is

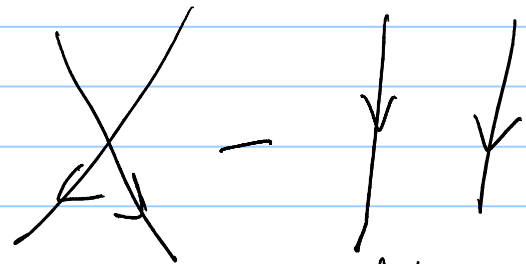


of Kirby II fame.



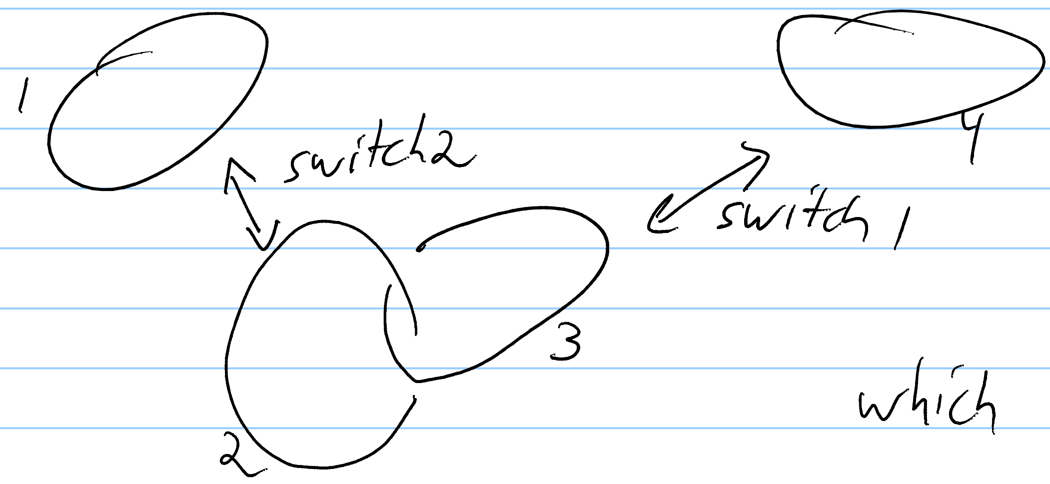
and likewise for Δ

Also:



"Finite type relative to switching knot components"

Funny Example: The Kontsevich integral is not-quite-finite-type relative to switching knot components:



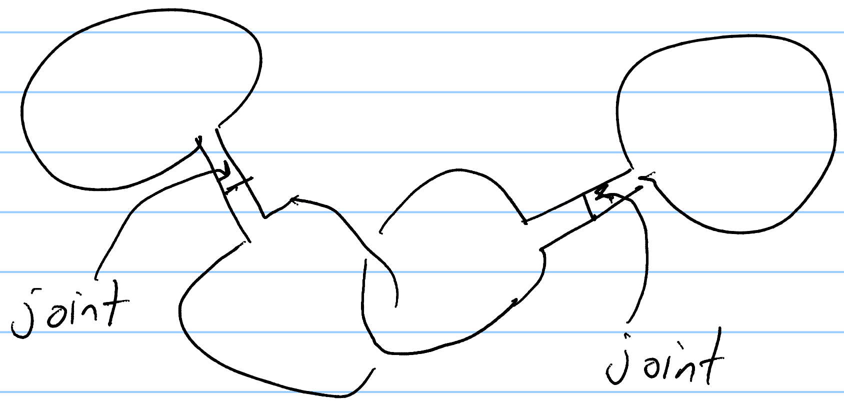
Two switches, but the degree 1 Z reads:

$\lambda_{23}-\lambda_{13}-\lambda_{24}+\lambda_{14}$

which is non-zero.

Question will "interdependent modifications" in the sense of Goussarov make things look better?

Goussarov could draw the above as



(has anybody bothered with a Goussarov theory for links/KTGs?)

(more or less; really, these dumbells only move one component from one place to another. To be complete, each dumbell should be doubled, so as two move two components at the same time)